CRITICAL ANALYSIS OF MOVEMENT FOR PARTS OF THE BODY OF STOCK - SOME QUESTIONS DYNAMIC MOVEMENT OF THE WHEELSET

KRITICKÁ ANALÝZA POHYBU ČASTÍ PODVOZKU KOĽAJOVÝCH VOZIDIEL - NIEKTORÉ DYNAMICKÉ OTÁZKY POHYBU DVOJKOLESIA

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SUMMARY:

A mobile wagon wheelset with a transverse dislocated geometric centre movement was considered in this study. In an absence of a nosing was obtained the law of the wheelset motion in the direction of the rolling-stock motion. A conclusion for a motion in a transverse direction instability was made.

KEYWORDS: *wagon wheelset, absence of a nosing, rolling-stock.*

We examine the movement of mobile wagon wheelset, assuming absence of nosing. In fig. 1 the wheelset is in general position with a transverse dislocated geometric centre. We introduce two coordinate systems.

The beginning of the first coordinate system O_{xyz} is $O \equiv C$ (geometric centre of the wheelset), when the two wheels roll along circumferences of their rim profile with identical radii $r_Д = r_Л = r$.

We connect the second coordinate system $O'_{x'y'z'}$ to the wheelset, as the beginning is in the geometric centre and *x´,y´,z´* are the main central axes of inertia for it. In a case of occurred deviation $y_c>0$, possible situation for the wheelset is the position, in which the instantaneous centre of rotation along *y´* axis coincides with the tangent point of the right wheel and rail.

It is known [1] that when we have movement in straight section the right wheel will perform pure rolling and the left - rolling with slip $v_{Ax} \neq 0$.

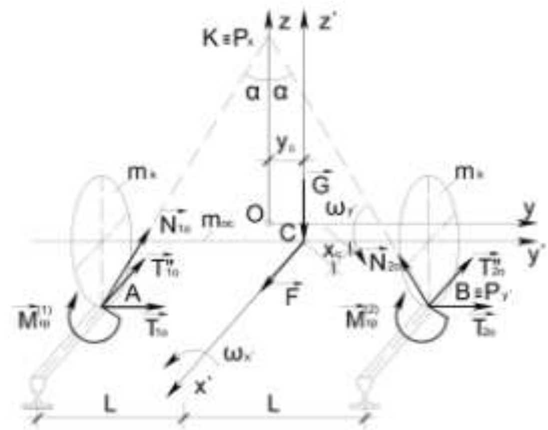

Fig.1

The forces acting on wheelset are:

- $\vec{G}$ - Gravity Force;
- $\vec{N}_{10}$ and $\vec{N}_{20}$ - Normal Pressure Forces, applied in contact points A and B between the wheels and rails, perpendicular to the surfaces of the rims;
- $\vec{T}_{10}$ and $\vec{T}_{20}$ - Friction Forces in transverse direction;

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- $\vec{F}$ - Driving Force, applied in point C along wheelset movement;
- $\vec{T}_{10}^{II}$ and $\vec{T}_{20}^{II}$ - Friction Forces in longitude direction;
- $\vec{M}_{fr}^{(1)}$ and $\vec{M}_{fr}^{(2)}$ - Resisting Moment.

For the kinetic energy of the wheelset can be written:

$$T=\frac{1}{2}\left[(2m_k+m_{oc})\dot{y}_C^2+J_{x'}^{(k)}\omega_{x'}^2+(2m_k+m_{oc})\dot{x}_C^2+J_{y'}^{(k)}\omega_{y'}^2\right] \qquad (1)$$

where:

- m_k and m_{oc} are the masses of the wheels and axis;
- $J_{x'}^{(k)}$ and $J_{y'}^{(k)}$ are Moments of Inertia of the wheelset related to *x´* and *y´* axes.

The kinematic characteristics, when we assume absence of nosing are defined in [1]:

$$\omega_{y'}=\frac{v_{Cx}}{r_Д}=\frac{\dot{x}_C}{r+z_B}=\frac{\dot{x}_C}{r+y_C tg\alpha};$$
$$v_{Ax'}=\frac{2\dot{x}_C y_C tg\alpha}{r+y_C tg\alpha} \qquad (2)$$

$$\begin{cases} y_A=y_B=y_C; \\ z_B=y_B tg\alpha \Rightarrow \dot{z}_B=\dot{y}_B tg\alpha; \\ \omega_{x'}=\dfrac{\dot{z}_B}{L}=\dfrac{\dot{y}_C tg\alpha}{L}; \\ tg\beta=\dfrac{z_B}{L},\ z_B<<L \quad \Rightarrow \quad \beta\approx tg\beta=\dfrac{y_B}{L}tg\alpha=\dfrac{y_C}{L}tg\alpha. \end{cases} \qquad (3)$$

After substitution of (2) and (3) in (1) the following expression for the kinetic energy of the wheelset is obtained:

$$T=\frac{1}{2}\left[\begin{array}{l}(2m_k+m_{oc})\dot{y}_C^2+J_{x'}^{(k)}\dfrac{y_C^2\,tg^2\alpha}{L^2}+ \\ +(2m_k+m_{oc})\dot{x}_C^2+J_{y'}^{(k)}\dfrac{x_C^2}{(r+y_C tg\alpha)^2}\end{array}\right] \qquad (4)$$

In the accepted links and accordingly established kinematic dependencies, only the parameters x_c and y_c are independent. For summarized coordinates we choose:

$$q_1=x_C; \qquad q_2=\beta=\frac{y_C}{L}tg\alpha;$$
$$\dot{q}_1=v_{Cx}=\dot{x}_C; \qquad \dot{q}_2=\omega_{x'}=\dot{\beta}. \qquad (5)$$

Applying the principle of the possible velocities, we could express the following:

$$Q_1\dot{q}_1+Q_2\dot{q}_2=F\dot{x}_C-T_1^{II}v_A-(M_{fr}^{(1)}+M_{fr}^{(2)})\omega_{y'}-Gy_C\omega_{x'}+(T_{10}^{I}+T_{20}^{I})\frac{L}{\sin\alpha}\omega_{x'} \qquad (6)$$

From (6) for the summarized forces is found:

$$\begin{cases} Q_1=F-\dfrac{2T_1''y_C tg\alpha-M_{fr}^{(1)}-M_{fr}^{(2)}}{r+y_C tg\alpha}; \\ Q_2=-Gy_C+(T_{10}'+T_{20}')\dfrac{L}{\sin\alpha}. \end{cases} \qquad (7)$$

From the static consideration of the wheelset in [1] is known:

$$T_{10}'+T_{20}'=\frac{Gy_C\sin\alpha}{L} \qquad (8)$$

Therefore from (8) and the second equation of (7), it is seen that Q_2=0.

To formulate the differential equations for the wheelset movement we apply equations of Lagrange, which in the case are two and have the following expression:

$$\begin{cases} \dfrac{d}{dt}\dfrac{\partial T}{\partial\dot{x}_C}-\dfrac{\partial T}{\partial x_C}=Q_1; \\ \dfrac{d}{dt}\dfrac{\partial T}{\partial\dot{\beta}}-\dfrac{\partial T}{\partial\beta}=Q_2. \end{cases} \qquad (9)$$

For the partial derivatives in (9) we have:

$$\begin{cases} \dfrac{\partial T}{\partial\dot{x}_C}=\left[(2m_k+m_{oc})+\dfrac{J_y^{(k)}}{r+y_C tg\alpha}\right]\dot{x}_C; \\ \dfrac{\partial T}{\partial x_C}=0; \\ \dfrac{\partial T}{\partial\dot{\beta}}=\left[(2m_k+m_{oc})\left(\dfrac{L}{tg\alpha}\right)+J_{x'}^{(k)}\right]\dot{\beta}; \\ \dfrac{\partial T}{\partial\beta}=0. \end{cases} \qquad (10)$$

We substitude (10) in (9) and after reductions we obtain:

$$\begin{cases} \left[(2m_k + m_{oc}) + \dfrac{J_{y'}^{(k)}}{r + y_C tg\alpha} \right] \ddot{x}_C = \\ = F - \dfrac{2T''_{10} y_C tg\alpha - M_{fr}^{(1)} - M_{fr}^{(2)}}{r + y_C tg\alpha}; \\ \left[J_{x'}^{(k)} + (2m_k + m_{oc}) \dfrac{L}{tg\alpha} \right] \ddot{\beta} = 0. \end{cases} \qquad (11)$$

The expression in the square brackets in the second differential equation in (11) is constant, therefore its I-st and II-nd integrals are:

$$\dot{\beta} = C_1, \qquad \beta = C_1 t + C_2. \qquad (12)$$

From (12) it can be concluded that the wheelset movement is unstable with respect to $q_2 = \beta$ coordinate.

From the kinematic characteristics (2) we express $y_c = \frac{\beta L}{tg\alpha}$ then substituting the expression in the first differential equation (11) we obtain:

$$\left(m + \frac{J_{y'}^{(k)}}{r + \beta L} \right) \ddot{x}_C = \frac{F - 2T''_{10}\beta L - M_{fr}^{(k)}}{r + \beta L} \qquad (13)$$

where:

- $m = 2m_k + m_{oc}$ - mass of the wheelset;
- $M_{fr}^{(k)} = M_{fr}^{(1)} + M_{fr}^{(2)}$ - Summarized Resisting Moment.

Let's solve analytically the differential equation (13):

$$\ddot{x}_C = \frac{F - 2T''_{10}\beta L - M_{fr}^{(k)}}{m(r + \beta L) + J_{y'}^{(k)}}. \qquad (14)$$

Substituting $\beta = C_1 t + C_2$ from (12) in (14) we have:

$$\ddot{x}_C = \frac{F - 2T''_{10}LC_1 t - 2T''_{10}LC_2 - M_{fr}^{(k)}}{mr + mLC_1 t + mLC_2} \qquad (15)$$

where:

$a = -2T''_{10}LC_1$; $b = F - 2T''_{10}LC_2 - M_{fr}^{(k)}$;

$c = mLC_1$; $d = m(r + LC_2)$

For the I-st integral of (15) we perform the designated actions and we obtain:

$$\dot{x}_C = \left[\frac{c}{a} t + \frac{c}{a}\left(\frac{bc}{a} - d \right) \ln(ct + d) \right] + C_3 \qquad (16)$$

The obtained expression (16) is the Law of Velocity Change $\dot{x}_c$ and is denoted as follows:

$$\dot{x}_C = -\frac{mLC_2}{2T''_{10}LC_1} t + \frac{mLC_2}{2T''_{10}LC_1} \left[\frac{(F - 2T''_{10}LC_2 - M_{fr}^{(k)})mLC_1}{2T''_{10}LC_1} + m(r + LC_2) \right] \times \ln(mLC_1 t + m(r + LC_2)) + C_3. \qquad (17)$$

For II-nd integral of (15), we have:

$$x_C = \frac{ct^2}{2a} + \left(\frac{bc}{a} - d \right) \frac{1}{a} \int \ln(ct + d)\, dt + C_3 t + C_4$$

$$x_C = \frac{ct^2}{2a} + t\ln(ct + d) - t + \frac{d}{c}\ln(ct + d) + C_3 t + C_4 \qquad (18)$$

The obtained expression (18) is the Law of the Wheelset Movement x_c= f(t). We have its final expression after substituting a, b, c and d:

$$x_C = -\frac{mt^2}{4T''_{10}} + \left[\ln(mLC_1 t + m(r + LC_2)) - 1 + C_3 \right] t + \frac{r + LC_2}{LC_1} \ln(mLC_1 t + m(r + LC_2)) \qquad (19)$$

Conclusion

The Law for the wheelset movement in the direction of the rolling stock movement is derived as a result of this study. A conclusion about the instability of movement in transverse direction, which would cause impulse forces when the flanges have a contact with the rail threads is made.

LITERATURE

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