SUMMARY:

A mobile wagon wheelset with a transverse dislocated geometric centre movement was considered in this study. In an absence of a nosing was obtained the law of the wheelset motion in the direction of the rolling-stock motion. A conclusion for a motion in a transverse direction instability was made.

KEYWORDS: wagon wheelset, absence of a nosing, rolling-stock.

We examine the movement of mobile wagon wheelset, assuming absence of nosing. In fig. 1 the wheelset is in general position with a transverse dislocated geometric centre. We introduce two coordinate systems. The beginning of the first coordinate system \( O_{xyz} \) is \( O \equiv C \) (geometric centre of the wheelset), when the two wheels roll along circumferences of their rim profile with identical radii \( r_L = r_R = r \).

We connect the second coordinate system \( O'_{x'y'z'} \) to the wheelset, as the beginning is in the geometric centre and \( x', y', z' \) are the main central axes of inertia for it. In a case of occurred deviation \( y_c > 0 \), possible situation for the wheelset is the position, in which the instantaneous centre of rotation along \( y' \) axis coincides with the tangent point of the right wheel and rail. It is known [1] that when we have movement in straight section the right wheel will perform pure rolling and the left - rolling with slip \( v_{Ax} \neq 0 \).

![Fig. 1](image.png)

The forces acting on wheelset are:
- \( \vec{G} \) - Gravity Force;
- \( \vec{N}_{10} \) and \( \vec{N}_{20} \) - Normal Pressure Forces, applied in contact points A and B between the wheels and rails, perpendicular to the surfaces of the rims;
- \( \vec{T}_{10} \) and \( \vec{T}_{20} \) - Friction Forces in transverse direction;

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\[ \ddot{F} \] - Driving Force, applied in point C along wheelset movement;
\[ \ddot{T}_{10}^B \] and \[ \ddot{T}_{20}^B \] - Friction Forces in longitude direction;
\[ \dddot{M}_{fr}^{(1)} \] and \[ \dddot{M}_{fr}^{(2)} \] - Resisting Moment.

For the kinetic energy of the wheelset can be written:
\[ T = \frac{1}{2} \mathbf{m}_1 \mathbf{v}_1^2 + \frac{1}{2} \mathbf{m}_2 \mathbf{v}_2^2 + \frac{1}{2} \mathbf{m}_3 \mathbf{v}_3^2 + \frac{1}{2} \mathbf{m}_4 \mathbf{v}_4^2 \] \tag{1}

where:
- \( \mathbf{m}_1 \) and \( \mathbf{m}_2 \) are the masses of the wheels and axis;
- \( J_{x}^{(1)} \) and \( J_{y}^{(k)} \) are Moments of Inertia of the wheelset related to \( x' \) and \( y' \) axes.

The kinematic characteristics, when we assume absence of nosing are defined in [1]:
\[ \omega_y = \frac{v_{Cy}}{r_{z}} = \frac{x_C}{r + z_B} \frac{r + y_C \cos \alpha}{r + y_C \cos \alpha}; \]
\[ v_{Ax'} = \frac{2 \dot{x}_C x_C y_C \cos \alpha}{r + y_C \cos \alpha} \tag{2} \]

\[ y_C = y_A = y_C; \]
\[ z_B = y_A \cos \alpha \Rightarrow z_B = y_C \cos \alpha; \]
\[ x_C L \]
\[ \beta = \frac{z_B}{L}, \quad z_B < L \Rightarrow \beta = \frac{y_C}{L} \tag{3} \]

After substitution of (2) and (3) in (1) the following expression for the kinetic energy of the wheelset is obtained:
\[ T = \frac{1}{2} \left[ \mathbf{m}_1 \mathbf{v}_1^2 + \frac{1}{2} \mathbf{m}_2 \mathbf{v}_2^2 + \frac{1}{2} \mathbf{m}_3 \mathbf{x}_C^2 + \frac{1}{2} \mathbf{m}_4 \mathbf{y}_C^2 \right] \frac{1}{L} \tag{4} \]

In the accepted links and accordingly established kinematic dependencies, only the parameters \( x_C \) and \( y_C \) are independent. For summarized coordinates we choose:
\[ q_1 = x_C; \quad q_2 = \beta = \frac{y_C \cos \alpha}{L}; \]
\[ \dot{q}_1 = v_{Cx} = \dot{x}_C; \quad \dot{q}_2 = \omega_y = \dot{\beta}. \tag{5} \]

Applying the principle of the possible velocities, we could express the following:
\[ q_{10}, \dot{q}_{10} = p_{10}, \dot{p}_{10} - \dot{M}_{fr}^{(1)} - \dot{M}_{fr}^{(2)} \]

From (6) for the summarized forces is found:
\[ Q_1 = F - \frac{2T \gamma v \cos \alpha - M_{fr}^{(1)} - M_{fr}^{(2)}}{r + y_C \cos \alpha}; \tag{7} \]
\[ Q_2 = -G y_C + T_{10}' + T_{20}' \approx \frac{L}{\sin \alpha}. \]

From the static consideration of the wheelset in [1] is known:
\[ T_{10}' + T_{20}' = \frac{G y_C \cos \alpha}{L} \tag{8} \]

Therefore from (8) and the second equation of (7), it is seen that \( \dot{Q}_2 = 0 \).

To formulate the differential equations for the wheelset movement we apply equations of Lagrange, which in the case are two and have the following expression:
\[ \frac{d}{dt} \frac{\partial T}{\partial \dot{q}} - \frac{\partial T}{\partial q} = Q_1; \]
\[ \frac{d}{dt} \frac{\partial T}{\partial \dot{\beta}} - \frac{\partial T}{\partial \beta} = Q_2. \tag{9} \]

For the partial derivatives in (9) we have:
\[ \frac{\partial T}{\partial \dot{x}_C} = \left[ \mathbf{m}_1 \mathbf{v}_1^2 + \frac{1}{2} \mathbf{m}_2 \mathbf{v}_2^2 + \frac{1}{2} \mathbf{m}_3 \mathbf{x}_C^2 + \frac{1}{2} \mathbf{m}_4 \mathbf{y}_C^2 \right] \frac{1}{L} \dot{x}_C; \]
\[ \frac{\partial T}{\partial \dot{\beta}} = 0; \]
\[ \frac{\partial T}{\partial \dot{\beta}} = \left[ \mathbf{m}_1 \mathbf{v}_1^2 + \frac{1}{2} \mathbf{m}_2 \mathbf{v}_2^2 + \frac{1}{2} \mathbf{m}_3 \mathbf{x}_C^2 + \frac{1}{2} \mathbf{m}_4 \mathbf{y}_C^2 \right] \dot{\beta}; \tag{10} \]
\[ \frac{\partial T}{\partial \dot{\beta}} = 0. \]
We substitute (10) in (9) and after reductions we obtain:

\[
\begin{align*}
\ddot{\beta} &= \frac{F_m + m \omega_c}{r + y_c \tan \alpha} \\
\beta &= C_1 t + C_2.
\end{align*}
\]

From (12) it can be concluded that the wheelset movement is unstable with respect to \( q_2 = \beta \) coordinate.

From the kinematic characteristics (2) we express \( y_c = \frac{BL}{r \tan \alpha} \) then substituting the expression in the first differential equation (11) we obtain:

\[
\left( m + J''_{s, \text{k}}^r \right) \ddot{x}_c = \frac{F - 2T''_{10} \beta L - M''_{f_r}}{r + \beta L}
\]

where:
- \( m = 2m_k + m_c \) - mass of the wheelset;
- \( M''_{f_r} = M''_{f_r}^{(1)} + M''_{f_r}^{(2)} \) - Summarized Resisting Moment.

Let’s solve analytically the differential equation (13):

\[
\ddot{x}_c = \frac{F - 2T''_{10} \beta L - M''_{f_r}}{m \beta L + J''_{s, \text{k}}^r}.
\]

Substituting \( \beta = C_1 t + C_2 \) from (12) in (14) we have:

\[
\ddot{x}_c = \frac{F - 2T''_{10} \beta L - M''_{f_r}}{m \beta L + J''_{s, \text{k}}^r}.
\]

For the I-st integral of (15) we perform the designated actions and we obtain:

\[
x_c = \frac{c t + \left( \frac{b c}{a} - d \right) \ln t + d}{a} + C_3
\]

The obtained expression (16) is the Law of Velocity Change \( x'_c \) and is denoted as follows:

\[
x_c = \frac{m \ell_{C_2}}{2T''_{10} \beta L} \int_1^{t_{C_1}} \left( \frac{m \ell_{C_2}}{2T''_{10} \beta L} \right) \ln \ell_{C_1} + \ell_{C_2} + C_4
\]

For II-nd integral of (15), we have:

\[
x_c = \frac{c t^2}{2a} + \left( \frac{b c}{a} - d \right) \int_1^{t_{C_1}} \ln \ell_{C_1} + \ell_{C_2} + C_4
\]

The obtained expression (18) is the Law of the Wheelset Movement \( x_c = f(t) \). We have its final expression after substituting \( a, b, c \) and \( d \):

\[
x_c = \frac{m t^2}{4T''_{10}} \ln m L \ell_{C_1} + m \ell_{C_2} + C_4
\]

Conclusion

The Law for the wheelset movement in the direction of the rolling stock movement is derived as a result of this study. A conclusion about the instability of movement in transverse direction, which would cause impulse forces when the flanges have a contact with the rail threads is made.

LITERATURE