



## CRITICAL ANALYSIS OF THE MOTION OF RUNNING GEAR ELEMENTS OF ROLLING STOCK IN NONHOLONOMIC SETUP - A NONHOLONOMIC CONNECTION MODELLING DURING A ROLLING - STOCK AXLE SHAFT MOVEMENT

### KRITICKÁ ANALÝZA POHYBU ČASTÍ PODVOZKU KOĽAJOVÝCH VOZIDIEL - NONHOLONOMICKÉ MODELOVANIE POHYBU HRIADEĽA NÁPRAVY

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#### SUMMARY:

An axle shaft movement on a straight section with horizontal unevenness in nonholonomic formulation was researched in this study. Coiling was considered as complex movement, causing dynamic interference in rolling-stock motion. Nonholonomic connections of the wheel pair axle movement were worked out by speed projection of the point of contact between it and the rail.

**KEYWORDS:** movement, rail wheel, inertial forces, track movement, nonholonomic connection.

#### INTRODUCTION

The wagon stock axle is a basic part of the rolling wheel set. It directs its motion along the track and transfers all loads from the wagon to the rails and reversed. Therefore, a detailed evaluation and modeling of this important element of the wagon is needed to increase the safety movement of trains.

#### 1. NOSING OF WHEEL PAIR

Rolling-stock movement on a straight section is considered.

In Fig. 1 is shown a wheel pair with different circles of rolling of the two wheels ( $r_{\text{Л}} > r_{\text{П}}$ ).

The shown wheel pair has shifted centroid, transversely within a  $z_c$  distance from the axis of the track.

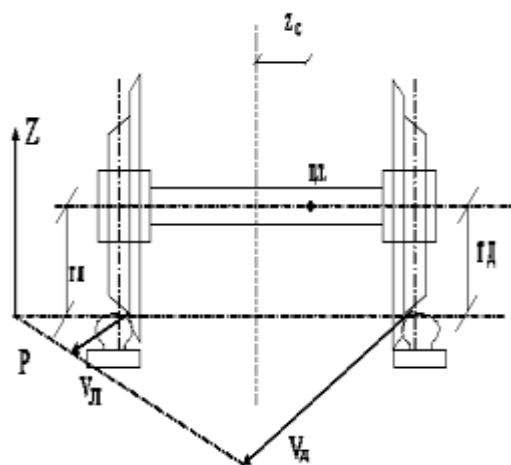


Fig.1

In another case due to technological reasons, caused by uneven wear-out of the wheels, the circles of rolling are different ( $r_{\text{Л}} \neq r_{\text{П}}$ ) even when the centroid of the wheel pair coincides with the axis of the path. In these conditions, the wheel pair will both move translationally

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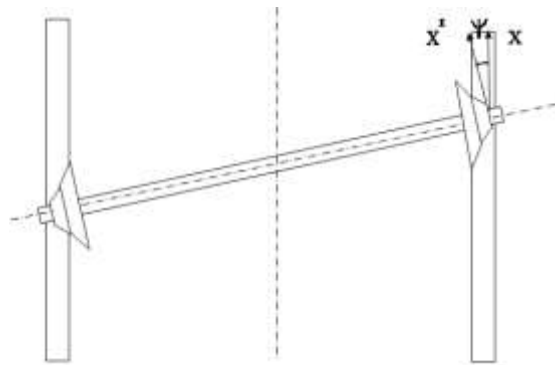
and rotate around the Z vertical axis, which intersects P – the instantaneous centre of velocities. That is because, the two wheels are fixed to the axis and have the same angular velocity  $\omega$ , as  $v_D > v_{\Pi}$  when  $r_D > r_{\Pi}$ .

A complex movement, called nosing is performed as a result of the simultaneous translational longitudinal to the track movement, the transversely dislocation and rotation around the vertical axis. It is problematic in terms of the wagon dynamics.

The large inertial forces that emerge cause both considerable pressure on the rail and discomfort of the passengers, result in adverse effects on the freight and sometimes impose limiting the speed of the train.

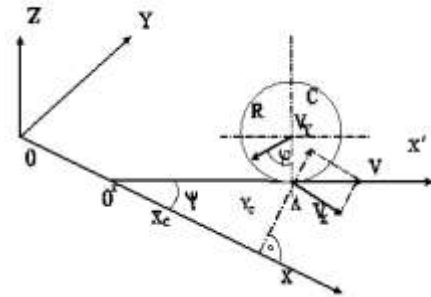
## 2. NONHOLONOMIC CONNECTIONS

Under nosing, the wheels of the running gears of a rolling stock move in nonholonomic setup.



**Fig.2**

In Fig. 2 is shown wheel pair, rotated around the vertical axis, intersecting the instantaneous centre of velocities.



**Fig.3**

Fig. 3 shows one of the wheels of the wheel pair, illustrated by the circle of rotation with radius  $R$ . If the rolling of the circle, situated in a vertical plane, is on scabbled horizontal plane, its position is defined by the coordinates of the centroid  $x_c$  and  $y_c$ , angle  $\Psi$ , between  $O^x$  axis and the track of the plane of the circle with the horizontal plane  $O_{xy} - O^x_{x'}$  and the angle  $\varphi$  when rolling of the circle. The circle has four degrees of freedom.

The nonholonomic connections, under which the movement is performed, are obtained by expressing the projections of the velocity  $v$  on the axis  $X$  and  $Y$ :

$$\begin{cases} v_x = v \cos \psi; \\ v_y = v \sin \psi. \end{cases} \quad (1)$$

Considering that:

$$v = R \frac{d\varphi}{dt}; \quad v_x = \frac{dx}{dt}; \quad v_y = \frac{dy}{dt}. \quad (2)$$

and substituting (2) in (1), we obtain nonholonomic connections as follows:

$$\begin{cases} dx = R \cos \psi d\varphi; \\ dy = R \sin \psi d\varphi. \end{cases} \quad (3)$$

Equation (3) is relation that connects the summarized velocities (respectively differentials) with the coordinates of the location. They are non-linear differential connections that cannot be integrated. Therefore the movement of the circle is with nonholonomic connections.

## LITERATURE

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